COMP 790: Stochastic Performance Modeling and Scheduling in Computer Systems

Homework 1: Probability Review

Released: August 18, 2022   Due: August 22, 2022

Instructions: This assignment is due at the start of class on the above due date. Collaboration is allowed, but please note your collaborators and turn in your own version of your answers. For more info on collaboration, see the course syllabus. Please utilize office hours if you get stuck! Office hours are Tuesday 10:00 - 11:30 and Thursday 4:00 - 5:30 in FB 336. All problems come from the course textbook unless otherwise stated.

Book Problems: 3.9, 3.11, 3.12, 3.13, 3.22, 3.23, 3.24, 3.25

Problem 1 (Sheldon Ross's Mouse Trap)**.** A mouse is trapped in a maze. Initially it has to choose one of two directions. If it goes to the right, then it will wander around in the maze for 3 minutes and will then return to its initial position. If it goes to the left, then with probability $\frac{1}{3}$, it will depart the maze after 2 minutes of traveling, and with probability $\frac{2}{3}$ it will return to its initial position after 5 minutes of traveling. Assume that the mouse is at all times equally likely to go to the left or the right. Let T denote the number of minutes that it will be trapped in the maze.

  (a) What is $\mathbb{E}[T]$?

  (b) What is $\mathrm{Var}(T)$?

Problem 2 (Darts)**.** We are given a line segment, $[0, 1]$. Kristy and Timmy each independently throw a dart uniformly at random within the line segment. What is the expected distance between Kristy's and Timmy's darts?